to flutter, so that the flutter modes H and Φ consisted of (h_1q_1, h_3q_3) and (ϕ_1q_1, ϕ_3q_3), respectively. In contrast, when the ply angle was −10 deg, all three modes contributed to flutter mode, so that H and Φ consisted of (h_1q_1, h_2q_2, h_3q_3) and (ϕ_1q_1, ϕ_2q_2, ϕ_3q_3), respectively. It can be concluded that modal interchanges can and will significantly alter the flutter speed and associated flutter mode of a composite wing in an uncharacteristic way, in which it is possible to observe sudden jumps or discontinuities in flutter speeds as a result of changing the ply orientations in a laminate.

Acknowledgments

The authors are grateful to Adam Sobey for many useful discussions. They are also grateful to one of the anonymous reviewers of Ref. 7, whose comments partly motivated this work.

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Minimum-State Approximation: A Pure Lag Approach

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Introduction

EXACT linear, time-invariant, finite state representation of the equations of motion for a flexible aircraft is, in general, rendered impossible by the presence of transcendental functions that arise in the description of the unsteady airloads acting on the aircraft. However, strong motivation exists for a linear, time-invariant, finite state representation because of the ease of solution of such systems as a consequence of the availability of efficient linear solvers. Rational function approximations (RFAs) to the unsteady airloads in the Laplace domain provide one method of allowing such a representation, albeit at the cost of an increased state vector dimension because of the appearance of additional states called aerodynamic lag states. These lag states are related to the basic system states through linear ordinary differential equations. Various studies[1–3] on rational function approximations have established the existence of a tradeoff between the accuracy of the fit and the number of additional (lag) states in the state vector. Of these, the minimum-state approximation,[2] termed the conventional minimum-state (CMS) approximation in this Note, appears to provide the best tradeoff, though at substantially increased computational costs. This Note is concerned with an improved form of the minimum-state approximation.

A major shortcoming of many existing RFAs[1–3] is their inability to fully isolate quasisteady and unsteady aerodynamic effects. In particular, the structure of these RFAs does not allow for the identification of any single coefficient as the quasisteady aerodynamic damping matrix. Further, while the form of these approximations seems to suggest that the first three terms in the approximation are completely representative of the quasisteady aerodynamics, this is not the case if one or more lag poles are included in the approximation. This observation is of considerable significance from the point of view of model order reduction through residualization techniques[4,5] and provides motivation for the development of a form of the approximation wherein the quasisteady and unsteady terms are decoupled. Further motivation for the isolation of quasisteady and unsteady aerodynamic terms comes from the need to have a form of the approximation that allows for easy adjustment of the first three coefficient matrices in the approximation to conform to data experimentally obtained from wind-tunnel tests or from computational fluid dynamics (CFD) codes, typically in the form of static and/or dynamic derivatives, which correspond to the quasisteady aerodynamic stiffness and quasisteady aerodynamic damping matrices, respectively. Panda[5] and Suryanarayan et al.[6] developed an extension of the RFA studied by Roger,[1] which allows for the separation of the unsteady aerodynamics into quasisteady terms and terms representative purely of the lag effects associated with the unsteady wake. This approximation was termed the pure lag approximation. The earlier approximations[1–3] were not amenable to such an interpretation. The pure lag approximation has been further extended by Mujumdar and Balan[7] to a multiple-order pole form. The advantages of the pure lag representation, coupled with the large saving in the number of aerodynamic lag states for a given fit accuracy afforded by the CMS approximation, make it a prime candidate for extension to the pure lag case. It is the aim of this Note to develop a pure lag minimum-state (PLMS) approximation and demonstrate its advantages compared with the CMS approximation.

Pure Lag Minimum-State Approximation

The CMS approximation to the matrix of generalized unsteady aerodynamic influence coefficients $Q(\bar{s})$ for unit dynamic pressure is described by the equation

$$Q(\bar{s}) \approx A_0 + A_1\bar{s} + A_2\bar{s}^2 + D(\bar{s}I - R)^{-1}E\bar{s} \tag{1}$$

where R is a diagonal matrix of lag poles, of dimension $N_a \times N_a$, D and E are, in general, nonsquare matrices of appropriate dimension, and $\bar{s}$ is the nondimensionalized Laplace variable $\bar{s} = \lambda s$, with $\lambda = b/U_\infty$, b being a reference length and U_∞ being the freestream velocity.

Received Nov. 6, 1996; revision received Sept. 15, 1997; accepted for publication Sept. 19, 1997.

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An examination of the first and second derivatives of the CMS approximation with respect to $\bar{s}$ illustrates the inability of this approximation to fully decouple quasisteady and unsteady aerodynamic terms. Differentiating Eq. (1) with respect to $\bar{s}$ yields

$$\frac{\partial Q}{\partial \bar{s}} \approx A_1 + 2\bar{s}A_2 + D(\bar{s}I - R)^{-1}[I - \bar{s}(\bar{s}I - R)^{-1}]E \quad (2)$$

whereas a second differentiation with respect to $\bar{s}$ yields

$$\frac{\partial^2 Q}{\partial \bar{s}^2} \approx 2A_2 + 2D(\bar{s}I - R)^{-2}[\bar{s}(\bar{s}I - R)^{-1} - I]E \quad (3)$$

In particular, the first and second derivatives of $Q(\bar{s})$ may be written at zero reduced frequency, in the form

$$\left.\frac{\partial Q}{\partial \bar{s}}\right|_{\bar{s}=0} \approx A_1 - DR^{-1}E \quad \left.\frac{\partial^2 Q}{\partial \bar{s}^2}\right|_{\bar{s}=0} \approx 2A_2 - 2DR^{-2}E \quad (4)$$

The first derivative of the unsteady aerodynamic matrix at zero reduced frequency is the quasisteady aerodynamic damping matrix, and twice the second derivative at zero reduced frequency is termed the aerodynamic inertia matrix, by analogy with the incompressible case.

An examination of Eqs. (1) and (4) reveals that the lag terms in the approximation contribute to the first and second derivatives of the unsteady aerodynamic matrix, and thereby to the quasisteady aerodynamic effects. The main disadvantage of the CMS approximation is its inability to decouple the quasisteady and pure lag terms. This disadvantage has important repercussions, particularly from the point of view of dynamic residualization.[6]

For the quasisteady aerodynamic damping matrix to be given as A_1 and the aerodynamic inertia matrix to be given as A_2, the terms $DR^{-1}E\bar{s}$ and $DR^{-2}E\bar{s}^2$ may be added to the right-hand side of Eq. (1). The resulting equation after simplification suggests an approximation of the form

$$Q(\bar{s}) \approx A_0 + A_1\bar{s} + A_2\bar{s}^2 + D(\bar{s}I - R)^{-1}E\bar{s}^3 \quad (5)$$

Equation (5) defines the PLMS approximation. The vector of aerodynamic lag states for this approximation is defined by the equation

$$\xi_a(\bar{s}) = (\bar{s}I - R)^{-1}E\bar{s}^2\xi(\bar{s}) \quad (6)$$

where $\xi(\bar{s})$ is the vector of generalized coordinates in the Laplace domain. The equations of motion of the aircraft may be written in the time domain in the form

$$(M - \lambda^2 P_{\text{dyn}}A_2)\ddot{\xi} + (C - \lambda P_{\text{dyn}}A_1)\dot{\xi} + (K - P_{\text{dyn}}A_0)\xi = \lambda P_{\text{dyn}}D\dot{\xi}_a + F(t) \quad (7)$$

where M, C, and K are the modal generalized mass, damping, and stiffness matrices, respectively; P_{dyn} is the dynamic pressure; and $F(t)$ is some external forcing function.

The method of computation of the coefficient matrix A_0 is identical for the CMS and PLMS approximations. The matrices A_1 and A_2 are, however, computed by different methods in the two approximations. In the CMS approximation, these matrices are computed either through constraining the real and imaginary parts of the approximation to be exactly equal to the tabulated values at, in general, two different reduced frequency locations,[2] or through inclusion as additional variables to be computed through the iterative nonlinear least-squares technique[8] used to compute the matrices D and E. The substantial increase in computational costs in the case of the latter alternative has prompted the investigation of techniques to reduce the cost of these computations.[8] It is significant to note that the PLMS approximation does away with the need for such techniques and also simplifies the incorporation of quasisteady aerodynamic data obtained through wind-tunnel tests or CFD codes into the approximation, because the computation of A_1 and A_2 in these approximations is carried out using the values of $Q(\bar{s})$ and its first and second derivatives at $\bar{s} = 0$, using the equations

$$A_0 = Q(0), \qquad A_1 = \lim_{\bar{s}\to 0}\frac{\partial Q}{\partial \bar{s}}, \qquad A_2 = \frac{1}{2}\lim_{\bar{s}\to 0}\frac{\partial^2 Q}{\partial \bar{s}^2} \quad (8)$$

A_0, A_1, and A_2 can thus be computed if the values of $Q(\bar{s})$ are known at enough points in the vicinity of $\bar{s} = 0$ for the accurate numerical evaluation of the derivatives occurring in Eq. (8). Thus, a small change in the form of the approximation leads to considerable simplification in the explicit incorporation of quasisteady aerodynamic parameters obtained through other techniques, such as wind-tunnel tests and/or CFD codes.

Results

The numerical example considered in the present study involves unsteady airloads typical of the longitudinal motion of a delta wing fighter aircraft, over a frequency range that includes the first three symmetric elastic modes of the aircraft, and is the same as the numerical example considered by Mujumdar and Balan.[7] The aircraft is modeled with seven degrees of freedom comprising two rigid modes, four elastic modes, and one symmetric control surface mode. The modal generalized unsteady airloads were computed using the doublet-lattice method for unit dynamic pressure assuming symmetric level flight at sea level, for $M = 0.9$, at 25 equally spaced reduced-frequency locations in the $\bar{k} = 0.0$–0.3 range. The matrix A_0 was computed as the value of the unsteady aerodynamic matrix at $\bar{k} = 0$, whereas the derivative computations required for the calculation of A_1 were carried out using a finite difference procedure. Converged values for the elements of A_1 were obtained for finite difference computations with a step size of 0.0005. The multiple-order pole pure lag RFA (Ref. 7) with $N_L = 4$ was used to compute A_2 for the present numerical example.

Figure 1 shows a comparison of the fit errors obtained using the CMS and PLMS approximations after 150 *D-E-D* iterations. The lag pole values in both cases were chosen as successive integral multiples of -0.2. Both approximations in Fig. 1 show comparable errors, thereby establishing the validity of the PLMS approximation. For the sake of comparison, Fig. 1 also shows fit errors obtained using the simple-pole pure lag rational function approximation,[7] whereby it is clear that the

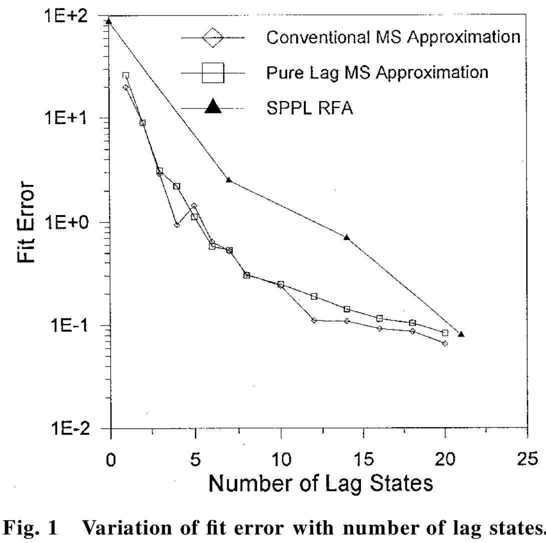

Fig. 1 Variation of fit error with number of lag states.

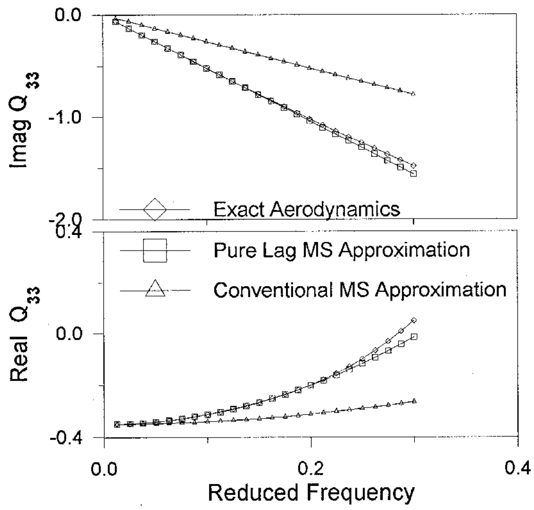

Fig. 2 Variation of three-term reconstruction for element Q_{33} with reduced frequency.

minimum-state approximations provide a much better tradeoff between fit accuracy and state vector dimension than other RFAs.

The nonmonotonic nature rate of fall of fit error with number of *D-E-D* iterations, as reported by Karpel,[2] was also observed in the case of the PLMS approximation. However, the two approximations required a different number of *D-E-D* iterations to arrive at the same level of fit error for the same number of aerodynamic lag poles. This phenomenon necessitates special care in comparing the two approximations. However, the first plateau in the curve of fit error vs number of *D-E-D* iterations was almost always reached before 150 iterations in both approximations.

To bring out the differences in the interpretation of A_0, A_1, and A_2 in the two approximations, the curve fits for both approximations were first carried out using six lag poles each. After computation of the coefficient matrices, the Q_{ij} were reconstructed using only the first three of these matrices, A_0, A_1, and A_2. As a typical illustration, Fig. 2 shows the variation of Q_{33} with reduced frequency $\bar{k}$ using such a reconstruction. It is obvious from Fig. 2 that such a reconstruction using the CMS approximation does not agree with the actual values, even over a limited range of reduced frequency, in contrast to that obtained using the PLMS approximation, which matches well with the frequency domain values. This is because the CMS approximation is consistent only in its totality and has component terms that lack physical consistency. However, in the PLMS approximation, the quasisteady and pure lag terms are decoupled. Similar trends were observed for other elements of the unsteady aerodynamic matrix as well. The pure lag curves in Fig. 2 also allow for the establishment of the domain of validity of the quasisteady approximation for various elements of the unsteady aerodynamic matrix. In particular, the quasisteady approximation is valid for element Q_{33} up to around $\bar{k} = 0.2$. The PLMS approximation thus provides a facility to progressively increase the range of validity of the approximation by the addition of more terms, starting from a one-term approximation. The CMS approximation does not provide this feature.

Conclusions

A PLMS approximation has been developed from the CMS approximation for unsteady aerodynamic loads. The new form of the approximation decouples quasisteady and pure lag terms and allows for direct incorporation of quasisteady wind-tunnel or CFD data into the approximation, in contrast to a more computationally elaborate method through incorporation of constraints, as carried out in the CMS approximation. It is also easier to carry out model order reduction through dynamic residualization in a consistent manner through the use of the PLMS approximation, because of the quasisteady aerodynamic damping matrix being characterized by a single coefficient matrix in the new approximation. Results obtained for the PLMS approximation demonstrate its utility as well as its advantages over the CMS approximation.

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Control of Leading-Edge Vortices with Suction

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Introduction

SEVERAL control techniques have been applied to control leading-edge vortices over delta wings at high angle of attack. The purposes of these control techniques are to influence the strength and structure of the vortices, to generate rolling moment, and to delay vortex breakdown. The application of suction offers advantages over other methods because of its simplicity. The earliest application of suction for vortex control is reported by Werle,[1] who demonstrated a delay of vortex breakdown by applying suction along the vortex axis. Parmenter and Rockwell[2] conducted similar experiments and described the transient response of vortices to suction.

Because the vorticity of the leading-edge vortices originates from the separation point along the leading edge, control of development of the shear layer by blowing/suction has been chosen as a control strategy in several investigations. Wood et al.[3] and Gu et al.[4] applied blowing and suction in the tangential direction along a rounded leading edge. These studies showed that a rounded leading edge can alter the location of separation

Received Oct. 6, 1997; accepted for publication Oct. 17, 1997.

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